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Laminar free-convective heat transfer from a vertical isothermal plate to glycerol with variable physical properties

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Abstract

The steady laminar boundary layer flow of glycerol along a vertical stationary isothermal plate is studied in this paper. The density, thermal conductivity and heat capacity of this liquid are linear functions of temperature but dynamic viscosity is a strong almost exponential function of temperature. The results are obtained with the numerical solution of the boundary layer equations. Both upward and downward flow is considered. The variation of μ with temperature has a significant influence on the wall heat transfer and a much stronger influence on wall shear stress. © 2002 Elsevier Science Inc. All rights reserved.

Keywords: Heated vertical plate; Free convection; Glycerol; Similarity

1. Introduction

It is known that many fluid properties (dynamic viscosity, thermal conductivity and heat capacity) are functions of temperature. For most fluids viscosity is more sensitive to temperature variations than heat capacity and thermal conductivity. Liquids can undergo strong viscosity variations which are usually well described by exponential laws. On the other hand, heat capacity and thermal conductivity for gases and liquids are well represented by linear functions of temperature.

The earliest known theoretical treatment of variable property effects on free convection along a vertical isothermal plate is the perturbation analysis of Hara (1954) for air. There have been numerous subsequent similar investigations in the literature. The reader can find many relevant works in Carey and Mollendorf (1980) and Kakac et al. (1985). In the present paper we focus on glycerol for the following reasons. The density, thermal conductivity and heat capacity of this liquid are almost linear functions of temperature but its dynamic viscosity is a strong function of temperature. These functions are given by Cuckovic-Dzodzo et al. (1999) for temperatures between 20 and 60 °C. For an isothermal plate with temperature 60 °C and ambient glycerol temperature 20 °C the Prandtl number varies from 825 to 10 785. The glycerol viscosity is about 1000 times that of water at 20 °C. Although glycerol has been used as the working fluid in some previous works (Cuckovic-Dzodzo et al., 1999; Jeevaraj and Patterson, 1992; Pittman et al., 1999; Selak and Lebon, 1997), to the best of the author's knowledge, glycerol has not been used in the classical problem of natural convection along a vertical isothermal surface with large temperature differences.

The boundary layer equations with variable fluid properties were solved by a method described by Patankar (1980). The finite difference method is used with primitive coordinates x, y and a space marching procedure is used in x direction with an expanding grid. Calculations were made on a DEC ALPHA 7000 computer using quadruple precision accuracy. The accuracy of the method was tested comparing the results with those of the classical free convection problem (constant viscosity and thermal conductivity and linear relationship between density and temperature, see Gebhart (1973)). More information about the equations, the boundary conditions and the solution procedure may be found in Pantokratoras (2001).

2. Results and discussion

In the similarity method commonly used in free convection over vertical surfaces, the following functions

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and variables are used. The nondimensional stream function $f(n)$, the similarity variable n, the local Grashof number Gr_x and the nondimensional velocity f'. These quantities are well known and can be found in the literature (see for example Pantokratoras, 2001). The most important quantities for this problem are the wall heat transfer and the wall shear stress defined as

$$\theta'(0) = \frac{x}{T_0 - T_a}\left[\frac{Gr_x}{4}\right]^{-1/4}\left(\frac{\partial T}{\partial y}\right)_{y=0}, \quad (1)$$

$$f''(0) = \frac{\rho_0 x^2}{\mu_0\sqrt{2}}[Gr_x]^{-3/4}\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad (2)$$

where T_0 and T_a are the plate and ambient temperatures and ρ_0 and μ_0 are the the glycerol density and dynamic viscosity at the wall.

For a given ambient glycerol temperature results were produced for different temperature differences between the plate and the ambient glycerol. For example, for $T_a = 20$ °C results were produced for $T_0 = 60$, 50, 40, 30 and 21 °C ($\Delta T = 40$, 30, 20, 10 and 1 °C). In addition, except of the usual problem of an upward moving fluid, results have been produced for downward flow (fluid cooling). In this case the plate temperature was lower than that of the ambient glycerol. For example, for $T_a = 60$ °C results were produced for $T_0 = 20$, 30, 40, 50 and 59 °C ($\Delta T = 40$, 30, 20, 10 and 1 °C). For each case the Prandtl number has been calculated at film temperature $(T_0 + T_a)/2$. The correspondence between the Prandtl number and the glycerol temperature is as follows:

T (°C)	60	50	40	30	20
Pr	825	1263	2310	4696	10 785

In Fig. 1 the wall heat transfer $-\theta'(0)$ is shown as function of the film Prandtl number Pr_f for different ambient temperatures. The dashed line corresponds to heat transfer for the classical problem of free convection along an isothermal vertical plate of a fluid with constant properties. Values of $-\theta'(0)$ for the classical problem are given by Gebhart (1973). For $Pr = 100$, 1000 and 10 000 the corresponding values are 2.1914, 3.9654 and 7.0852. Values for other Pr numbers have been calculated by the present method. The curves above the dashed line correspond to the upward glycerol flow and those below the dashed line to downward flow. Arrows show increasing ΔT. The points where the solid line curves meet the dashed line correspond to $\Delta T = 1$ °C. From this figure it is seen that as ΔT increases the wall heat transfer $-\theta'(0)$ decreases in the upward flow and increases in the downward flow. As ΔT increases the departure from the constant properties curve increases, too. For example, for $T_a = 20$ and $\Delta T = 10$ °C the wall heat transfer is 7.0100. For the same ambient temperature and $\Delta T = 40$ °C the wall heat transfer is 6.0854. The differences from the constant properties values (dashed line) are 8% and 25% respectively. From this figure it is also seen that the heat transfer with variable properties is higher than that of constant properties in the upward flow and lower than that of constant properties in the downward flow.

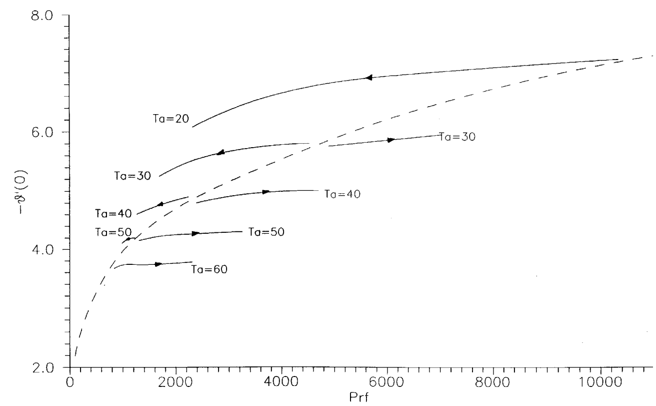

Fig. 1. Wall heat transfer as function of film Pr number for different ambient temperatures. Dashed line corresponds to constant properties. The curves above the dashed line correspond to upward flow and those below the dashed line to downward flow. Arrows show increasing ΔT.

In Fig. 2 the wall shear stress $f''(0)$ is shown for the same conditions of Fig. 1. Now the dashed line corresponds to wall shear stress for the classical problem of free convection along an isothermal vertical plate for a fluid with constant properties. Unfortunately there are no arithmetic results of $f''(0)$ for the classical problem and high Pr numbers. In the work of Gebhart (1985) for $Pr = 100$ a value of 0.2517 is given for $f''(0)$. Values for other Pr numbers have been calculated by the present method and are shown by the dashed line in Fig. 2. From this figure it is seen that as ΔT increases the wall shear stress increases for upward flow and decreases for downward flow. As ΔT increases the departure from the

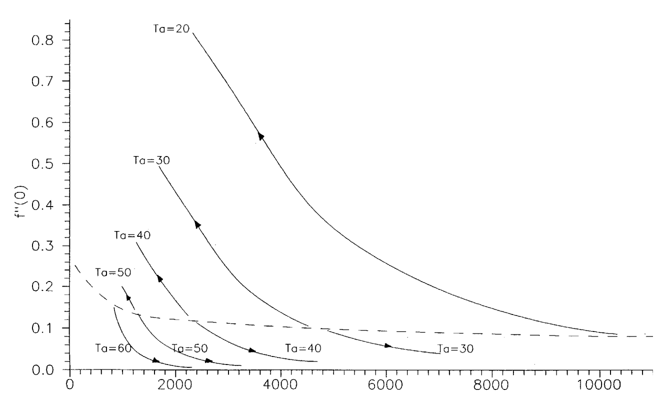

Fig. 2. Wall shear stress as function of film Pr number for different ambient temperatures. Dashed line corresponds to constant properties. The curves above the dashed line correspond to upward flow and those below the dashed line to downward flow. Arrows show increasing ΔT.

dashed line increases, too. For example, for $T_a = 20$ and $\Delta T = 10$ °C the wall shear stress is 0.1918. For the same ambient temperature and $\Delta T = 40$ °C the wall shear stress is 0.8171. The differences from the constant properties values are 118% and 581% respectively. The effect of temperature dependent viscosity on wall shear stress is much stronger than that of the wall heat transfer. Again the shear stress with variable properties is higher than that of constant properties in the upward flow and lower than that of constant properties in the downward flow.

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